Analogue Filters Design and Simulation

by

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Title page

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1. Abstract

This report describes how an active 4th order low pass filter is designed and simulated. The filter is constructed from some given specifications, where one of them is, that the filter need to have a Butterworth response. The procedure that is used to complete the design, are by using the common known Sallen and Key architecture. The values for the passive components are calculated, and the circuits are then simulated to reach a final conclusion which will describe the results of the simulations compared to the given specifications. The output of the report gives an ideal and an non-ideal approach of the assignment, where there in the non-ideal solution are used E12 values for the passive components and a non-ideal operational amplifier. This is done so that it is possible to build the circuit of the Butterworth filter for practical use.

Contents

2. Introduction

Today noise reduction, band limit of analogue signals and design of active filters is a very hot topic. It is very common to insert some kind of filter in for instance DSP applications prior to encoding and sampling processes, where a false input signal can be critical to the outcome on the output. Long before Op-Amps were available, electrical filters have been designed and used. That was when only passive components, such as resistors and capacitors could be used for the filters. Today the Op-Amps are low-cost and very reliable active devices, which is the reason for mainly using active filters today.

There are various types of active filters, where some of them are named Chebyshev, Bessel and Butterworth. The design process of the filters can consist of different types of architectures, where the two most common are named Sallen and Key, and Multiple Feedback (MFB). In this assignment the filter is an active low-pass filter, with a Butterworth response. The architecture that will be used is the Sallen and Key.

2.1. Butterworth filters

The figure 1 below shows the actual responses for three different types of filters the Bessel, Chebyshev and the Butterworth filter. The Butterworth response is the one that is we are going to concentrate about in this report.

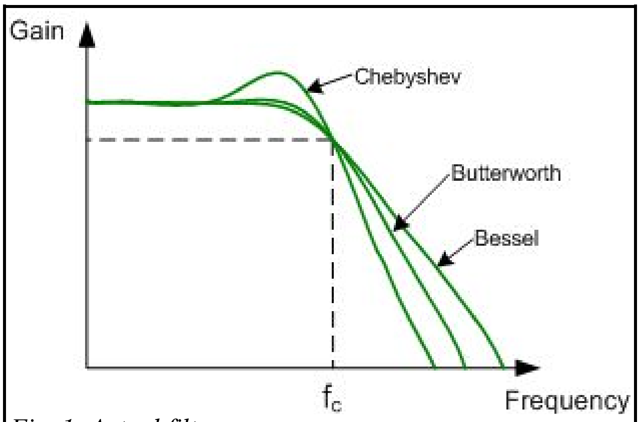

Fig. 1: Actual filter responses

The characteristics for the Butterworth filter response are:

- Maximal flat magnitude response filter.
- Optimised for gain flatness in the passband.
- -3dB at the cut-off frequency.
- -20dB per decade per order above the cut-off frequency.
- Transient response to a pulsed input shows moderate overshoot and ringing.

Butterworth polynomials:

Unlike other polynomials the Butterworth polynomials requires the least amount of work, because the frequency scaling scaling factor are always equal to one. The normalized polynomials for the fourth order low-pass Butterworth filter are shown below.

$$(s^2+0{,}765\,s+1)\cdot(s^2+1{,}848\,s+1)$$

Where: s = $j\omega$

To make it easier to design the different types of active filters there are made some filter coefficient tables for each type of responses.

3. Assignment specifications

1. **Filter response** = **Butterworth.**
2. **Filter type** = **Low-pass**
3. **Filter order** = **4^{th} order.**
4. **-3dB frequency** = **8kHz.**

The filter must be designed using modified Sallen and Key circuits. A visual indication of the magnitude response of the filter is shown in the figure 2 below:

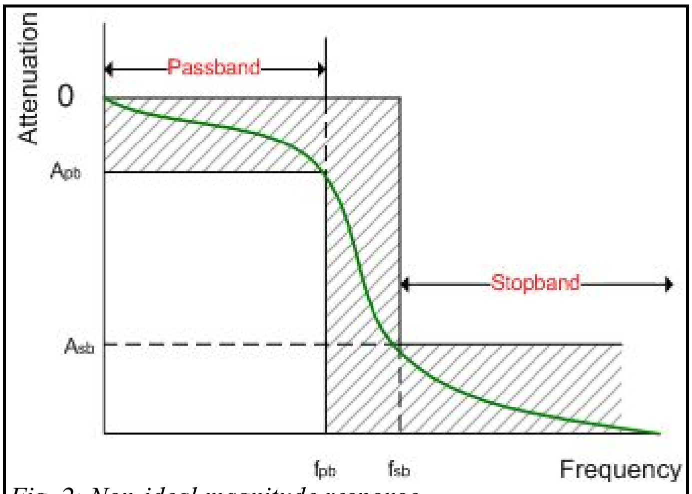

Fig. 2: Non-ideal magnitude response

The filter order of 4 means that the Butterworth filter now can be designed using two OP-amps for the Sallen-Key architectures. When the filter order is 4, the number of resistors and capacitors that is needed for the complete filter circuit are also 4 of each.

4. Sallen and Key

The Sallen and Key architecture is the easiest way to design an active filter. It is the design that uses the least number of components, and the equations are relatively straight forward. It has been discussed, analysed and reviewed in great depth on the web and in text books. The Sallen and Key is very often a preferred design compared to the MFB architecture because it does not invert the signal. Another advantage of using a Sallen and Key architecture, is that it is not sensitive to component variation at unity gain. A disadvantage is the high frequency response of the filter, that is limited by the frequency response of the amplifier.

4.1. Low-pass architecture

The ideal design of the Sallen and Key architecture as a fourth order, low-pass filter is shown in figure 3 using TINA. Look aside from the component values, which will be calculated to the correct values shortly. From TINA the general ideal transfer function for the fourth order circuit can be derived. With this transfer function it is possible to calculate the values of the passive components. Simplified formulas derived from the transfer function, will for that purpose be used. -The transfer functions denominator will be used later on to calculate the poles of the filter.

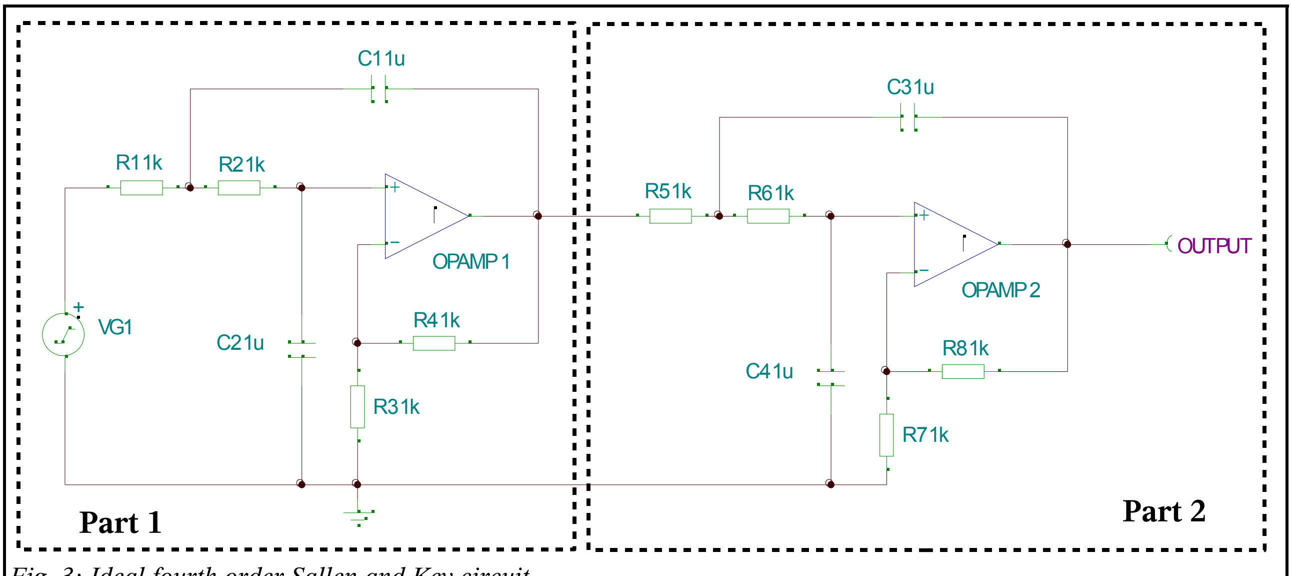

Fig. 3: Ideal fourth order Sallen and Key circuit

The filter are divided up in two second order Sallen and Key filters, where the values of the components will be calculated individually for the two parts and then cascaded.

4.2. Calculations part 1

Chosen values:

- C_1, C_2 = 2,2nF.
- R_3 = 47kΩ.

Known value:

- 2K = 0,765

R_1 is calculated using the cut-off frequency formula:

$$f_c = \frac{1}{2\cdot\pi\cdot R_1\cdot C_1} \Rightarrow R_1 = \frac{1}{2\cdot\pi\cdot f_c\cdot C_1} = \frac{1}{2\cdot\pi\cdot 8k\cdot 2{,}2n} = 9{,}04k \sim \underline{\underline{R_1 = 10k\Omega}} \qquad (E12\,value)$$

And $R_1 = \underline{\underline{R_2 = 10k\Omega.}}$

The gain factor is calculated:

$$3 - Av0 = 2K \Rightarrow Av0 = 3 - 2K \Rightarrow$$

$$Av0 = 3 - 0{,}765 = \underline{\underline{2{,}235}}$$

R_4 is calculated using the gain formula:

$$K = \frac{R_3 + R_4}{R_3} \Rightarrow R_4 = (K-1)\cdot R_3 \Rightarrow$$

$$R_4 = (2{,}235 - 1)\cdot 47k = 58k \sim \underline{\underline{56k\Omega}} \qquad (E12\,value)$$

Control calculation for the cut-off frequency with the E12 values:

Since the values for the R_1 and R_2 , C_1 and C_2 are the same, only R and C references for these components will be used in this formula.

$$f_c = \frac{1}{2\cdot\pi\cdot R\cdot C} = \frac{1}{2\cdot\pi\cdot 10k\cdot 2{,}2n} = \underline{\underline{7{,}23kHz}}$$

Control calculation for the gain with the E12 values:

$$K = \frac{R_3 + R_4}{R_3} = \frac{47k + 56k}{47k} = \underline{\underline{2{,}191}}$$

4.3. Calculations part 2

The resistors R_5, R_6 as well as the capacitors C_3, C_4 are set to be the same values as R_1 and R_2, C_1 and C_2, cause the cut-off frequency in part 2 of the circuit still has to be at the 8kHz. There are though a difference between the two circuits, and that's the gain, hence the design is a fourth order filter. This means that there is also a difference in the gain resistors between the two circuits. Calculations for this are made below.

Chosen value:.

- $R_7 = 47k\Omega$.

Known value:

- 2K = 1,848

The gain factor is calculated:

$$3-Av0=2K \Rightarrow Av0=3-2K \Rightarrow$$

$$Av0=3-1{,}848=\underline{\underline{1{,}152}}$$

R_8 is calculated using the gain formula:

$$K=\frac{R_7+R_8}{R_7} \Rightarrow R_8=(K-1)\cdot R_7 \Rightarrow$$

$$R_8=(1{,}152-1)\cdot 47k=7{,}14k \sim \underline{\underline{6{,}8k\Omega}} \qquad (E12\,value)$$

Control calculation for the gain with the E12 values:

$$K=\frac{R_7+R_8}{R_7}=\frac{47k+6{,}8k}{47k}=\underline{\underline{1{,}145}}$$

Overall gain with the ideal values:

$$K_{total}=Av0_{Part\,1}\cdot Av0_{Part\,2}=2{,}235\cdot 1{,}152=2{,}575=\underline{\underline{8{,}22\,dB}}$$

Overall gain with the non-ideal E12 values:

$$K_{total}=K_{Part\,1}\cdot K_{Part\,2}=2{,}191\cdot 1{,}145=2{,}509=\underline{\underline{7{,}99\,dB}}$$

4.4. Calculations of the poles

From the semi-symbolic AC-transfer function (derived from TINA), the poles of the Sallen and Key filter can be derived. This is done with the help of Mathcad where the transfer function is inserted. The s in the denominator, that indicates the poles of the circuit can then be found by letting Mathcad calculate on the equation. The result for the Sallen and Key pole calculation are shown below.

Poles for the ideal Sallen and Key circuit:

$$0=1+5{,}2\cdot 10^{-5}\cdot s+1{,}35\cdot 10^{-9}\cdot s^2+2{,}06\cdot 10^{-14}\cdot s^3+1{,}56\cdot 10^{-19}\cdot s^4$$

$$s=\begin{pmatrix} -46{,}881\,k\pm j18{,}767\,k\,\dfrac{rad}{sec} \\ -19{,}144\,k\pm j46{,}338\,k\,\dfrac{rad}{sec} \end{pmatrix}$$

Poles for the non-ideal Sallen and Key circuit:

$$0=1+5{,}86\cdot 10^{-5}\cdot s+1{,}69\cdot 10^{-9}\cdot s^2+2{,}84\cdot 10^{-14}\cdot s^3+2{,}34\cdot 10^{-19}\cdot s^4$$

$$s=\begin{pmatrix} -42{,}536\,k\pm j16{,}358\,k\,\dfrac{rad}{sec} \\ -18{,}147\,k\pm j41{,}572\,k\,\dfrac{rad}{sec} \end{pmatrix}$$

These pole outputs are a strong indication of the filters response, and whether the filter is stable or unstable. Later it is indicated if the simulation results are corresponding to the calculation results. In a practical matter of speaking, the pole points can give the designer of an active filter a very good indication on how the filter could be optimized if needed.

4.5. Simulations

Simulations are made using TINA with calculated ideal, and the non-ideal E12 values of the components. The circuits shown below in figure 4, indicates how the the Sallen and Key are made in TINA to get the ideal and non-ideal output simulation results that can be compared.

The figures on the following pages, shows how the output response are with an ideal OP-amp and the non-ideal OP-amp uA741. These can then be compared with the calculations illustrated above for Sallen and Key filter architecture.

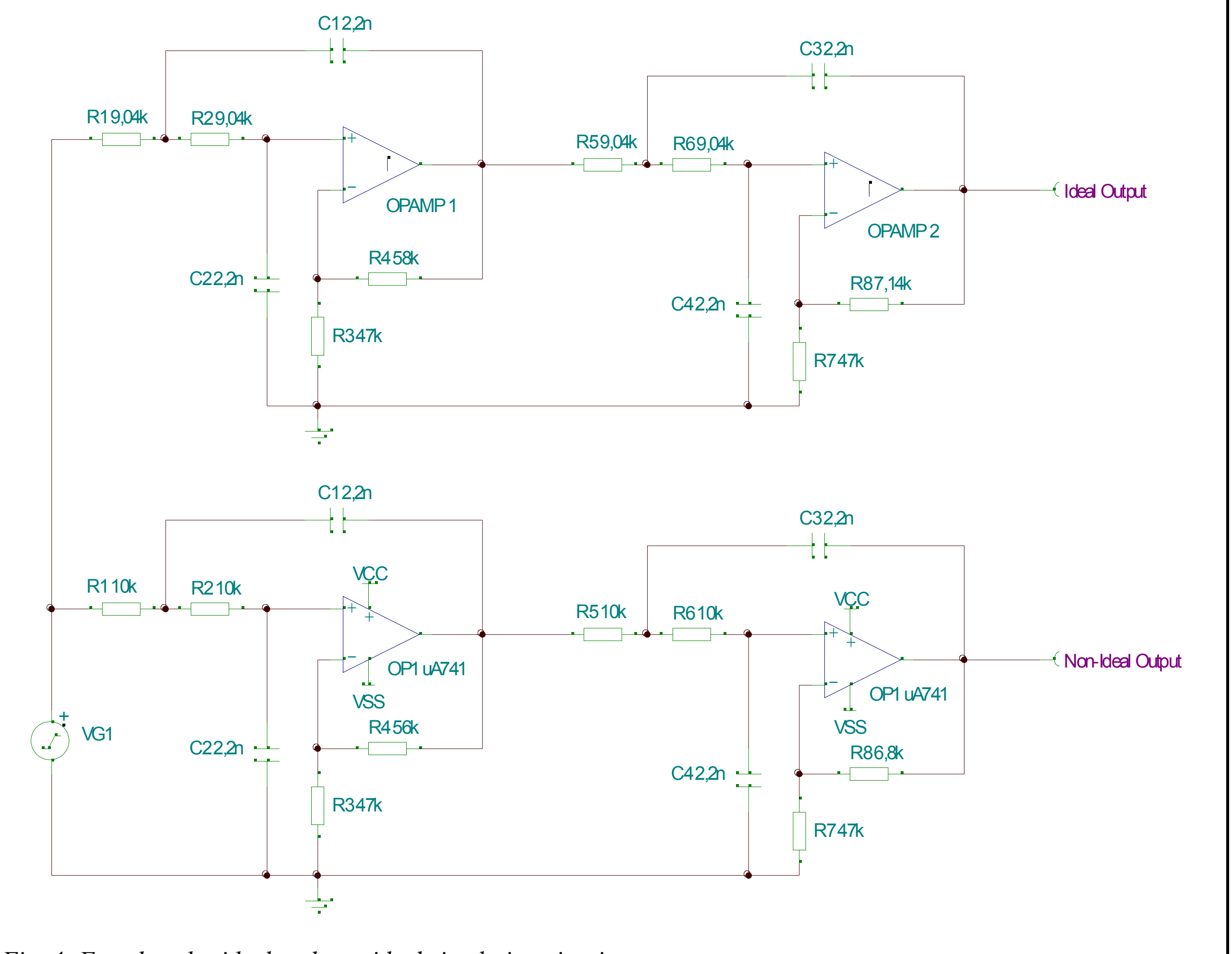

Fig. 4: Fourth order ideal and non-ideal simulation circuits

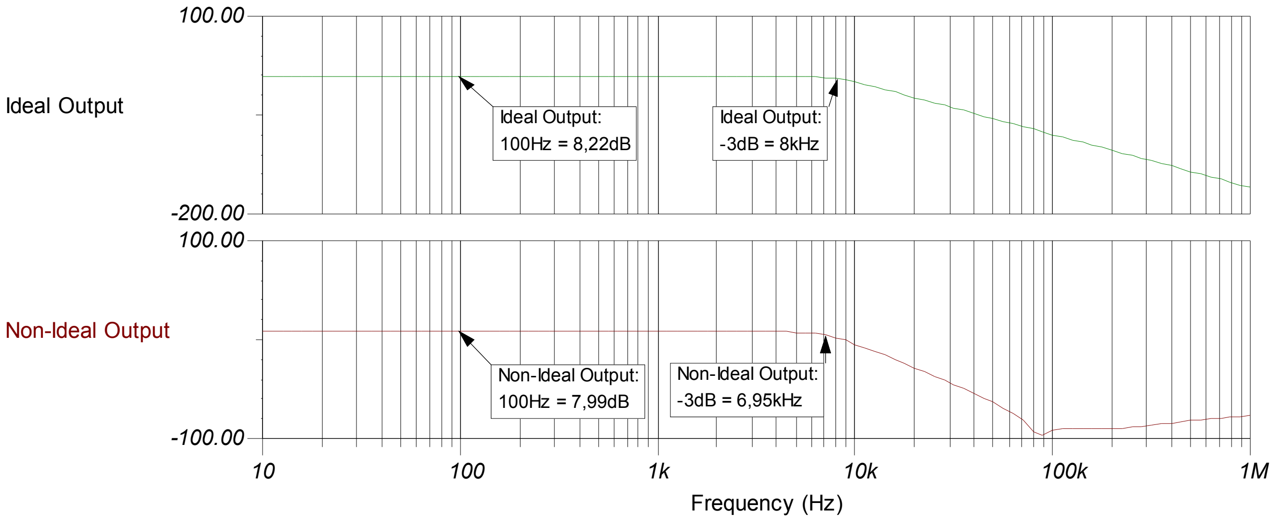

Fig. 5: Sallen-Key gain vs. frequency response

From the figure 5, it is possible to see the comparison with ideal and non-ideal circuits in their gain vs. Frequency response. The ideal response are like the calculations made before. The non-ideal on the other hand has a very different response. The cut-off frequency are not like the one shown in the control calculations where the E12 values are used for the resistors. This is however a result of the tolerance relationship at 5% and the OP-amps characteristics that TINA are using. And it is also possible to see that the OP-amp is not ideal, by looking at the non-ideal output of the graphs, that clearly indicates that with the Sallen-Key architecture, it is not useful above a frequency of 90kHz.

Hence this is a 4^{th} order filter with a Butterworth response, the stop-band attenuation will then be 80dB per decade, for the complete circuit. Usually there are 20dB per decade per filter order (with Butterworth response) above the cut-off frequency. Below are the measured values from TINA, where it is indicated the differences in frequencies, between the ideal and non-ideal circuits of one decade in the stop-band.

- **The ideal output:** -80dB = 10kHz-100kHz.
- **The non-ideal output:** -80dB = 10kHz-72,45kHz.

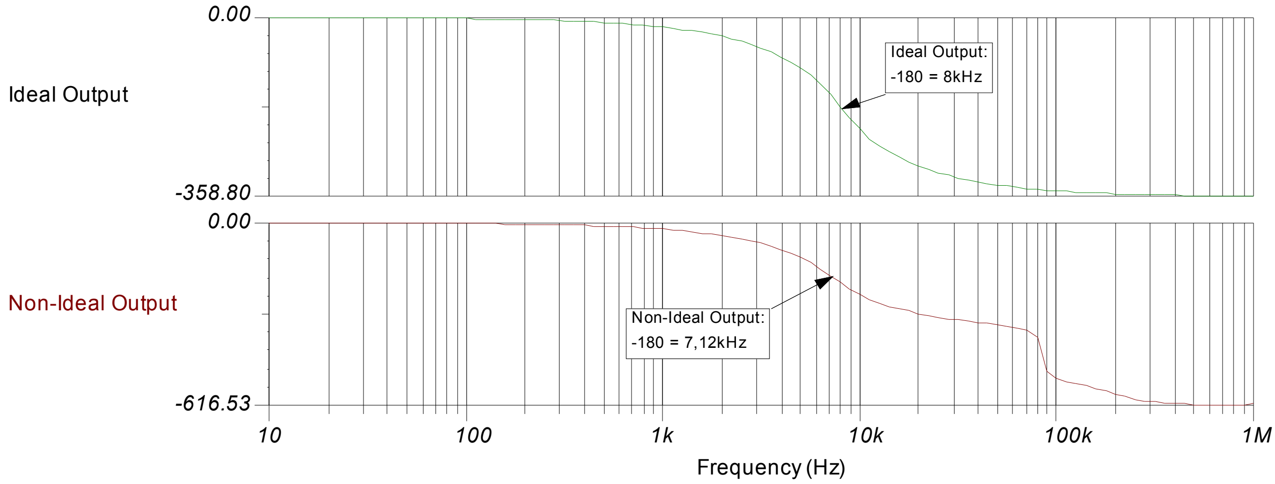

Fig. 6: Sallen-Key gain-phase response

The figure 6 indicates the gain-phase relationship for the 4th order Butterworth filter. For each pole the filter consists of, there is a phase shift of -45 degrees. This means, that the phase between the input to the output will be lagging 180 degrees at the cut-off frequency. Again the ideal output matches the calculations, and the non-ideal output are close to the control calculation.

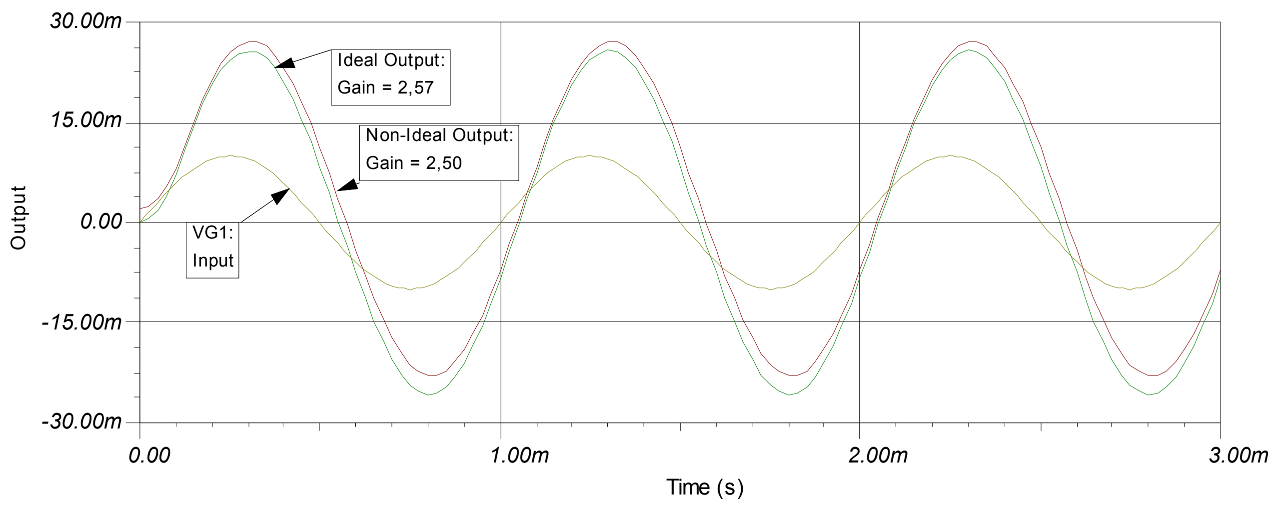

Fig. 7: Sallen-Key filter gain

With the transient output results from TINA (figure 7), the overall gain can be derived for the two circuits. The gain response matches the calculations for the ideal and non-ideal perfectly. The generator VG1 are set to give a frequency of 1kHz with an amplitude at ±10mV.

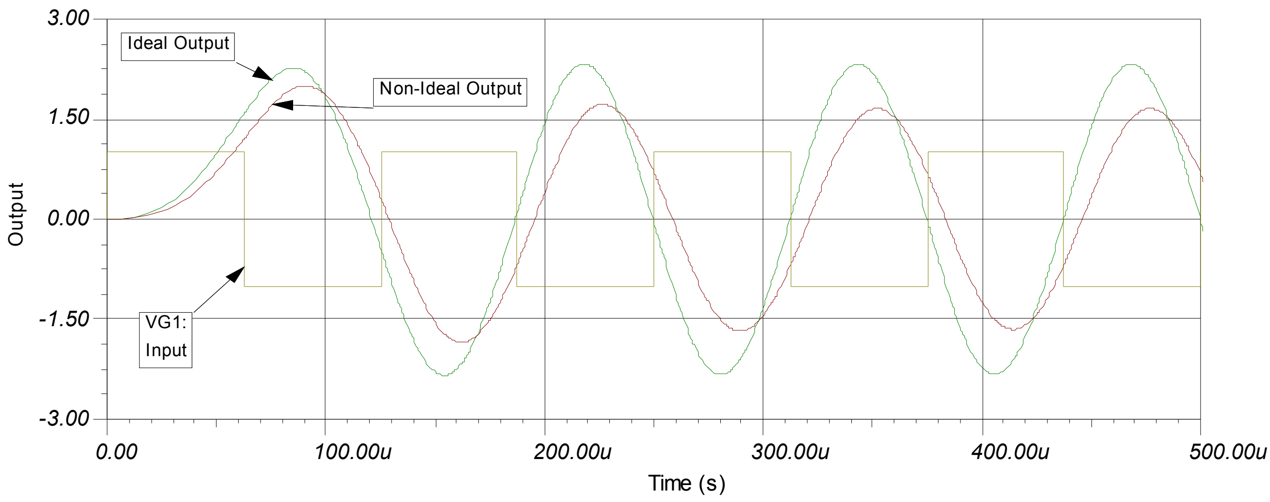

Fig. 8: Sallen-Key squares in - sinusoidal out

With the VG1 on the input of the filter, set to generate squares at 8kHz and an amplitude of±1V, the output now shows another neat function of the Butterworth filter. It is as shown in figure 8, possible to convert squares from the input to the output to a sinusoidal signal. This is a feature that the filter consists of above the cut-off frequency. Below the cut-off frequency (with squares on the input) the output response of the filter will show the overshoot and ringing, which are one of the characteristics of the Butterworth filter response. Note that the phase shift in figure 8 still are 180 degrees between the input to the output.

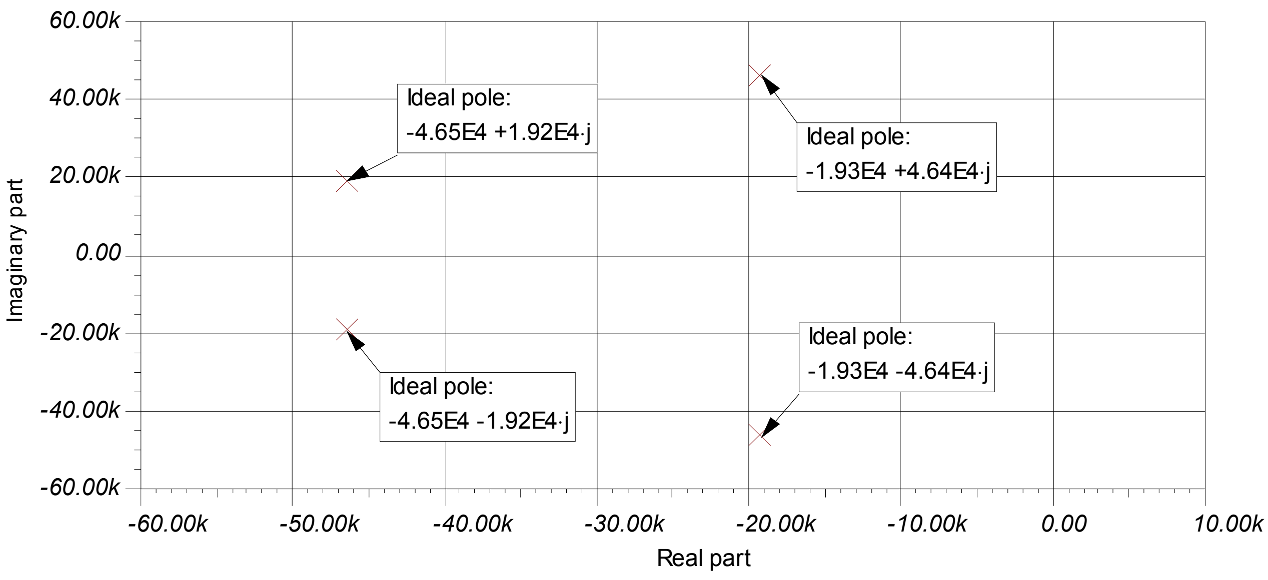

Fig. 9: Sallen-Key ideal pole plot

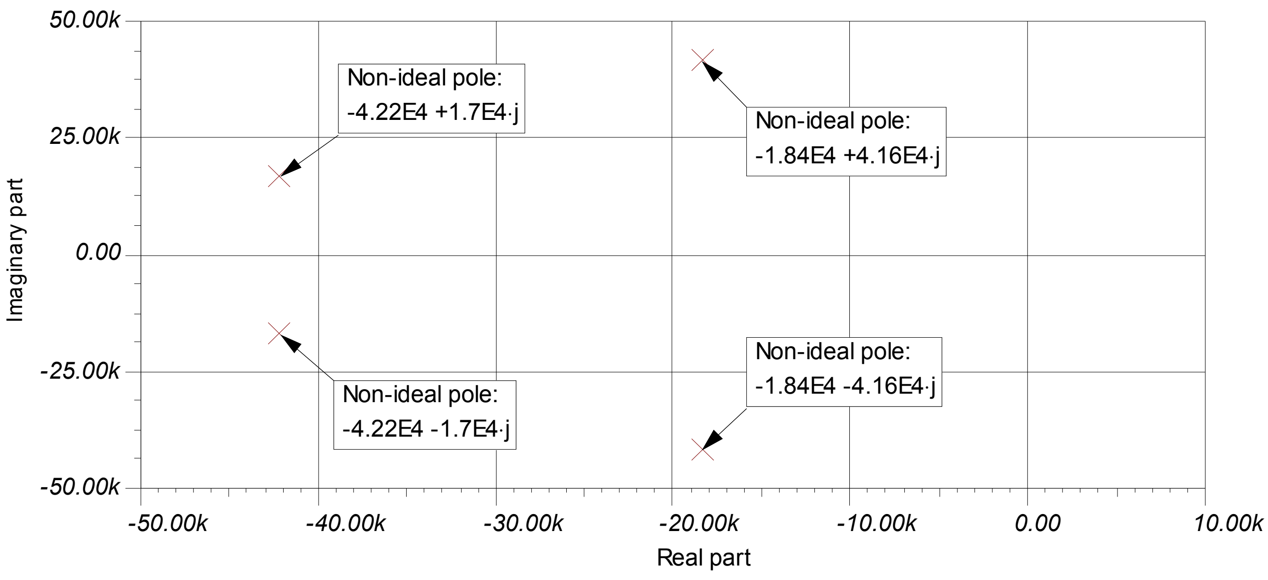

Fig. 10: Sallen-Key non-ideal pole plot

The pole plot simulations clearly states that there is a good relationship to the calculations. However there are still a slight difference when simulating the ideal circuit and its pole calculations. Note that it is still 5% in tolerance for the resistors and capacitors, so its not totally an ideal simulation. Since it is not possible to make TINA simulate the pole plot with an non-ideal OP-amp (the uA741), an ideal OP-amp is inserted in that circuit instead, referring to the simulation circuits in figure 4.

5. Improvements

There are various ways to improve the 4^{th} order Butterworth filter. One could be to make the gain of the filter adjustable by inserting an extra OP-amp at the output, where the gain can be adjusted with a potentiometer. An other way to improve the filter to give a better output response is to insert more precise components, which could be the passive resistors and capacitors, that ought to be changed to values with tolerances of 1%. Also other architectures than the Sallen-Key could be more useful, like the Multiple Feedback architecture (MFB). One of the advantages using the MFB topology is that it is less sensitive to component variations and has a superior high-frequency response compared to the Sallen and Key topology. Although the MFB architecture has one more component than the Sallen and Key, this filters response has better a stop-band rejection.

6. Conclusion

The active filter design and simulation assignment has been completed successfully. To make an active filter is not a new subject, but it has been designed with a different approach in this assignment. The previously project with the design of an active filter has been with Laplace calculations, where the method for completing this assignment where with more simplified formulas. That has made the interest for this assignment grow, and further more build active filters in the future, cause I have now experienced that there are more than one approach, and tools for designing these type of filters. The research I have been doing for this project has also been different. By reading a lot of application notes and books related to making Butterworth filters, there was a lot of stuff that came in quite handy for this assignment and also gave a lot second thoughts in designing active filters.

The capacitors for the non-ideal circuit have been selected within the E12 row so that it is practically possible to build the circuit. Of course the capacitors have to be in a special material to get the best results. For these applications metallized polycarbonate capacitors is a good choice (cause usual capacitors as ceramic capacitors can cause errors to the filter circuit) where the tolerance is at 1%. The resistors have been selected after the same principle as the capacitors (E12 values) , but with a slight difference in the tolerance which is at 5%. If better results should be achieved, which means that the ideal calculations results should be closer to non-ideal results, a change in the resistor tolerance could be a method for achieving this.

The Sallen-Key architecture has proven once again that the use of it, can give a very successful result. With the calculations of the ideal Sallen-Key architecture compared with the non-ideal results it can be concluded that they are leaning up quite close next to each other.

Another neat function that the active Butterworth filter has, is that it can convert square signals to sinusoidal signals between the input and the output. If a really good sinus is needed, this is possible to derive by making the filter order number higher. This method of converting a square signal to a sinusoidal are seen in a lot of new micro processors today like for instance the PSoC micro processors where the internal contents consists of “building blocks” where active filters is a possibility.

Overall this assignment has been interesting and has given some new tools on how to design active filters and a good understanding of the different types of architectures.

Carsten Kristiansen

7. References

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7.2. Literature

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7.3. Software tools

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